

The structure of projective indecomposable modules for A_n , $n \leq 12$

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Abstract

This article gives the structure of all projective indecomposable modules in blocks with non-cyclic defect group for $n \leq 11$, and almost all in the case of $n = 12$, leaving four simple modules for $p = 2$ and one simple module for $p = 3$.

In the representation theory of the symmetric and alternating groups, blocks with abelian defect group are, in some sense, well understood. The decomposition numbers are not known in general, the Ext^1 matrices are not known in general, the structure of the projectives are not known in general, but things are better than when the defect group is non-abelian. A lack of explicit examples is of no help, of course, and the purpose of this short paper is to provide researchers with a few more examples of Ext^1 -matrices and the socle structures of the projective indecomposable modules for A_n for $n \leq 12$. This bound is largely imposed on the author by the fact that A_{12} is already difficult to do via computer without advanced methods. Indeed, it is not the construction of the PIMs but analysing their socle structure that is the real issue, taking many times longer than their construction.

Although we are mainly interested in non-abelian defect groups – so $p = 2, 3$ – we include the Ext^1 -matrices and PIM structures for $p = 5$ as well, since they might be of use to people. For $p \geq 7$ the defect group is cyclic and the structure of all blocks are known by the theory of Brauer trees, as is the case for $p = 5$ and $n \leq 9$. We omit the description of all projectives that lie in blocks of defect 0 or 1.

If we look through the literature, we can find the following cases where the projective indecomposable modules are already described. There might well be others, but I cannot find them at the moment.

n	$A_n, S_n, p = 2$	$A_n, p = 3$	$S_n, p = 3$
$n = 7$	Erdmann [3]	Scopes [5]	Scopes [5]
$n = 8$	Benson [1]	Scopes [5]	Scopes [5]
$n = 9$	Benson [2]	Siegel [6]	Tan [8]
$n = 10$	All here	Tan [8]	Tan [8]
$n = 11$	All here	Three modules by Tan [7], rest here	Six modules by Tan [7], rest here
$n = 12$	All but four here	All but one here	This paper

Notice that for $p = 2$ we simply induce from A_n to S_n , replacing a module M in the diagram for each projective by the self-extension of M . (This also allows us to go backwards, of course.) For S_{10} and $p = 3$, the projectives are described in [8], and those for A_{10} can be obtained from them by restriction, although we reproduce them here for ease of reference. For $p = 5$ the blocks have weight 2, and are completely understood by work of Scopes [5]. We give the projectives here just to make life easier for people who want them.

1 A_{10} and S_{10} , characteristic 2

We firstly compute the projective indecomposable modules for the alternating group A_{10} , and then induce them to S_{10} to produce the projectives there.

There are seven simple modules in the principal block of S_{10} (and A_{10}) and they are below.

λ	$\dim(D^\lambda)$	Factors of S^λ
(10)	1	1
(9, 1)	8	8, 1
(6, 4)	16	16, 48, 26
(8, 2)	26	26, 8, 1
(7, 3)	48	48, 26, 1
(6, 3, 1)	198	198, 16, 48, 1, 26 ²
(5, 3, 2)	200	200, 198, 16, 26, 8, 1 ²

The non-principal block that does not have defect zero has two characters, $S^{(7,3,1)} = D^{(7,2,1)}$ of degree 160 and $D^{(5,4,1)}$ of degree 128.

Here are the projectives for the alternating group A_{10} , which we can then induce to S_{10} . For completeness we list them as well.

1	8
8 26 200	1
1 1 1 48 198	26 200
8 8 26 26 26 200	1 1 48
1 1 1 1 1 1 16 16 16 48	8 8 26
8 8 8 26 26 26 26 26 198 200	1 1 1 16 16
1 1 1 1 1 1 1 16 16 48 48 48 198	8 26 26 26 200
8 8 8 26 26 26 26 26 26 26 198 200 200 200	1 1 1 48 48 198
1 1 1 1 1 1 1 1 1 16 16 16 48 48 198	8 8 26 26 26 198 200
1 8 8 8 8 8 26 26 26 26 26 26 48 198 200 200	1 1 1 1 1 16 16 16
1 1 1 1 1 1 1 1 1 8 16 16 16 16 16 26 48 48 198	1 8 8 26 26 26 200
1 1 8 8 8 16 26 26 26 26 26 26 26 200 200	1 1 1 16 48 48 198
1 1 1 1 1 1 16 16 26 48 48 48 198	8 26 26 26 200
8 8 8 26 26 26 26 48 198 200	1 1 1 16
1 1 1 1 1 16 16 26 48	8 8 26 198
8 8 16 26 26 198 200 200	1 1 16 48
1 1 1 1 1 1 16 26 48	26 200
8 26 198 198 200	1
1	8

16	26
26	1 16 48 198
48 198	8 26 26 26
26 26 200	1 1 1 16 16 48 198
1 1 1 16	8 26 26 26 26 26 200
8 8 26	1 1 1 1 1 16 48 48 198
1 1 16 16 48	8 8 8 26 26 26 26 200
26 26 26 200 200	1 1 1 1 1 1 1 16 16 16 48 48
1 1 1 48 48 198	8 8 8 26 26 26 26 26 198 200 200
8 8 8 26 26 26 198	1 1 1 1 1 1 1 16 16 16 48 48 198
1 1 1 1 1 16 16 16 16	1 8 8 8 26 26 26 26 26 26 26 198 200 200
1 8 26 26 26 26 200	1 1 1 1 1 1 1 16 16 16 16 48 48 198
1 1 48 48 48 198	1 8 8 8 26 26 26 26 26 26 198 200
8 26 26 26	1 1 1 1 16 16 16 48 48 48 198
1 1 16 16 16 198	1 8 26 26 26 26 26 200
1 8 26 26 200	1 1 16 16 48 198 198
1 16 48 198	1 8 26 26 26
26 200	1 16 48 198
16	26
48	198
26	26
1 16	1 16
8 26	26
1 48 198	48
26 26 200	1 26
1 1 1 16 48	1 200
8 8 26 26	1 8 198
1 1 16 16 48	1 8 16 26
1 26 26 26 200	1 16 26 200
1 1 48 48 198	1 26 48 198
8 8 26 26 198	8 26 48
1 1 1 16 16 16	1 16 26
1 26 26 26 200	1 26
1 48 198	8 16 48 198
8 26	1 26 26
1 16	16 198 200
26 48	1 1 26
48	198
	200

Here are the symmetric group ones.

1
 1 8 26 200
 1 1 1 8 26 48 198 200
 1 1 1 8 8 26 26 26 48 198 200
 1 1 1 1 1 1 8 8 16 16 16 26 26 26 48 200
 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 48 198 200
 1 1 1 1 1 1 1 8 8 8 16 16 26 26 26 26 26 26 48 48 198 198 200
 1 1 1 1 1 1 1 8 8 8 16 16 26 26 26 26 26 26 26 48 48 48 198 198 200 200 200
 1 1 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 26 26 48 48 198 198 200 200 200
 1 1 1 1 1 1 1 1 1 8 8 8 8 16 16 16 26 26 26 26 26 26 26 48 48 198 198 200 200
 1 1 1 1 1 1 1 1 1 8 8 8 8 8 16 16 16 16 16 26 26 26 26 26 26 26 48 48 198 198 200 200
 1 1 1 1 1 1 1 1 1 1 8 8 8 8 16 16 16 16 16 16 26 26 26 26 26 26 26 26 48 48 198 200 200
 1 1 1 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 26 26 26 48 48 198 200 200
 1 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 26 48 48 48 198 198 200
 1 1 1 1 1 8 8 8 16 16 26 26 26 26 26 48 48 198 200
 1 1 1 1 1 8 8 16 16 16 26 26 26 48 198 200 200
 1 1 1 1 1 1 8 8 16 16 26 26 26 48 198 200 200
 1 8 26 198 198 200
 1

8	16
1 8	16 26
1 26 200	26 48 198
1 1 26 48 200	26 26 48 198 200
1 1 8 8 26 48	1 1 1 16 26 26 200
1 1 1 8 8 16 16 26	1 1 1 8 8 16 26
1 1 1 8 16 16 26 26 200	1 1 8 8 16 16 26 48
1 1 1 8 26 26 26 48 48 198 200	1 1 16 16 26 26 26 48 200 200
1 1 1 8 8 26 26 26 48 48 198 198 200	1 1 1 26 26 26 48 48 198 200 200
1 1 1 1 1 8 8 16 16 16 26 26 26 198 200	1 1 1 8 8 8 26 26 26 48 48 198 198
1 1 1 1 1 1 8 8 16 16 16 26 26 26 200	1 1 1 1 1 8 8 8 16 16 16 16 26 26 26 198
1 1 1 1 8 8 16 26 26 26 48 48 198 200	1 1 1 1 1 1 8 16 16 16 16 26 26 26 200
1 1 1 8 16 26 26 26 48 48 198 200	1 1 1 8 26 26 26 26 48 48 48 198 200
1 1 1 8 16 26 26 26 200	1 1 8 26 26 26 48 48 48 198
1 1 1 8 8 16 26 198	1 1 8 16 16 16 26 26 26 198
1 1 8 8 16 26 48 198	1 1 1 8 16 16 16 26 26 198 200
1 1 16 26 48 200	1 1 8 16 26 26 48 198 200
1 26 200	1 16 26 48 198 200
1 8	16 26 200
8	16

26
 1 16 26 48 198
 1 8 16 26 26 26 48 198
 1 1 1 8 16 16 26 26 26 48 198
 1 1 1 8 16 16 26 26 26 26 48 198 200
 1 1 1 1 1 8 16 26 26 26 26 48 48 198 200
 1 1 1 1 1 8 8 8 16 26 26 26 26 48 48 198 200
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 1 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 48 48 198 200 200
 1 1 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 26 48 48 48 198 198 200 200
 1 1 1 1 1 1 1 1 8 8 8 16 16 16 16 26 26 26 26 26 26 48 48 48 198 198 200 200
 1 1 1 1 1 1 1 1 8 8 8 16 16 16 16 26 26 26 26 26 26 26 48 48 198 198 200 200
 1 1 1 1 1 1 1 1 8 8 8 16 16 16 16 26 26 26 26 26 26 48 48 198 198 200
 1 1 1 1 1 8 8 8 16 16 16 26 26 26 26 26 26 48 48 48 198 198 200
 1 1 1 1 1 8 16 16 16 26 26 26 26 26 48 48 48 198 200
 1 1 1 8 16 16 26 26 26 26 26 48 198 198 200
 1 1 1 8 16 16 26 26 26 48 198 198
 1 1 8 16 26 26 26 48 198
 1 16 26 48 198
 26

48	198	200
26 48	26 198	1 200
1 16 26	1 16 26	1 8
1 8 16 26	1 16 26	1 8 16
1 8 26 48 198	26 48	1 16 26 200
1 26 26 48 198 200	1 26 48	1 26 48 200
1 1 1 16 26 26 48 200	1 1 26 200	1 8 26 48 198
1 1 1 8 8 16 26 26 48	1 1 8 198 200	1 1 1 8 16 16 26 198
1 1 8 8 16 16 26 26 48	1 1 8 8 16 26 198	1 1 1 8 16 16 26 26 200
1 1 1 16 16 26 26 26 48 200	1 1 8 16 16 26 26 200	1 1 8 26 26 48 198 200
1 1 1 26 26 26 48 48 198 200	1 1 16 26 26 48 198 200	1 1 8 26 26 48 198
1 1 8 8 26 26 48 48 198 198	1 8 26 26 48 48 198	1 1 8 16 26 26
1 1 1 8 8 16 16 16 26 26 198	1 8 16 26 26 48	1 1 8 16 26
1 1 1 1 16 16 16 26 26 26 200	1 1 16 26 26	1 8 26 48
1 1 26 26 26 48 198 200	1 8 16 26 48 198	1 26 48 200
1 8 26 48 198	1 8 16 26 26 48 198	1 1 16 26 200
1 8 16 26	1 16 26 26 198 200	1 1 8 16 198 200
1 16 26 48	1 1 16 26 198 200	1 8 16 198 200
26 48 48	1 1 26 198	1 16 200
48	198	200

This is the largest symmetric group where we are able to write the projectives like this, since the projective

cover of the trivial module is getting too wide for the page; thus we will just write down the projectives for the alternating groups.

2 A_{11} and S_{11} , characteristic 2

We will only give here the projectives for the principal block of the alternating group, leaving as an exercise placing another copy of a number directly above all numbers to generate the projectives for the symmetric group.

There are seven simple modules in the principal block of S_{11} (and A_{11}) and they are below.

λ	$\dim(D^\lambda)$	Factors of S^λ
(11)	1	1
(9, 2)	44	44
(6, 4, 1)	144	144, 164, 186, 198, 1
(7, 4)	164	164, 1
(8, 2, 1)	186	186, 44, 1
(7, 3, 1)	198	198, 186, 164, 1^2
(5, 4, 2)	416	416, 144, 186, 198, 44, 1^2

The non-principal block has 2-core $(2, 1)$, and has modules as follows for the alternating group.

λ	$\dim(D^\lambda)$	Factors of S^λ
(10, 1)	10	10
(6, 5)	$16_1 \oplus 16_2$	32, 100
(8, 3)	100	100, 10
(6, 3, 2)	848	848, 100, 32, 10
(5, 3, 2, 1)	$584_1 \oplus 584_2$	1168, 848, 32^2 , 100^2 , 10^3

Here are the projectives for the principal 2-block of the alternating group A_{11} , which we can then induce to S_{11} , if that is what you need.

1
 44 164 198 198 416
 1 1 1 1 1 186 186
 44 44 144 164 164 198 198 416 416
 1 1 1 1 1 1 1 186 186 186
 44 44 44 44 144 144 164 164 164 198 198 198 198 416 416
 1 1 1 1 1 1 1 1 1 1 186 186 186
 44 44 44 44 44 144 144 164 164 164 186 198 198 198 198 198 416 416 416
 1 1 1 1 1 1 1 1 1 1 1 186 186 186 186
 1 44 44 44 44 144 144 164 164 164 186 198 198 198 198 416 416 416
 1 1 1 1 1 1 1 1 1 1 144 164 186 186 186
 1 1 44 44 44 44 144 164 164 186 186 198 198 198 198 416 416
 1 1 1 1 1 1 1 1 144 164 186 186 198
 1 44 44 44 144 164 164 186 198 198 416 416
 1 1 1 1 1 1 1 144 164 186
 44 164 186 198 198 416

1

44	144	164
1	186	1
164 416	198	44 198
1 1 186	1	1 1 186
44 44 44 198 198	164 186 416	144 164 164 416
1 1 1 1 186	1 1	1 1 1 186
144 144 164 164 416 416	44 44 198	44 44 198 198 198
1 1 1 1 1 186 186	1 1 144	1 1 1 186
44 44 44 198 198 198 198	144 164 186 416	144 164 164 186 416
1 1 1 1 186	1 1 186 198	1 1 1 186
144 164 164 186 416 416	1 44 198	1 44 44 198 198
1 1 1 1 186	1 164 186 186	1 1 144 164 186
44 44 44 198 198	1 144 164 198	1 144 164 186 416
1 1 1 144	1 44 186	1 1 186 198
164 186 416	1 144 198	1 44 198
1 44	186 416	1 164
44	144	164

186	198	416
144 198	1 1 186	1
1 1 186	144 164 416	44 198
44 164 186 198	1 1 186	1 1
1 1 1 144	44 44 198 198 198	144 164 416
44 164 186 186 198 416	1 1 1 1 186	1 1 186
1 1 1 198	144 164 164 164 186 416 416	44 44 198 198
1 44 44 164 198 416	1 1 1 1 1 186	1 1 1 186
1 1 1 1 144 164 186 186	44 44 44 44 198 198 198 198	144 164 416 416
1 44 144 164 186 198 416	1 1 1 1 144 186	1 1 1 186
1 1 1 44 186 186 198 198	144 164 164 186 186 416 416	44 44 198 198
1 1 44 144 144 164 198	1 1 1 1 186 198	1 1
1 1 164 186 186 186 416	1 44 44 144 198 198	164 186 416
1 144 164 198 198	1 1 164 186 186	1 1
1 44 186 186	144 164 198 416	44 198
1 144 198	1 1 186	1 144
186	198	416

For the non-principal block of A_{11} , we need \mathbb{F}_4 for the two pairs of dual modules, of dimension 16 and 584. We start by giving the structure of the projectives over \mathbb{F}_2 for the modules 10, 100, 848, and the two irreducible but not absolutely irreducible modules of dimension 32 and 1168. We do this for ease of translation to the symmetric group case. We then will give the projective covers of 16 and 584 over \mathbb{F}_4 , choosing this labelling so that $\text{Ext}^1(584, 16)$ is non-zero (but $\text{Ext}^1(584, 16^*) = 0$).

10
100 100 848
10 10 32 32
100 100 100 848
10 10 32 32
100 100 100 848 1168
10 10 10 10 10 10 10 32 32 32
100 100 100 100 100 100 100 848 848 1168 1168
10 10 10 10 10 10 10 10 10 10 10 32 32 32 32 32 32
100 100 100 100 100 100 100 848 848 1168 1168
10 10 10 10 10 10 10 32 32 32
100 100 848 1168
10

32

100 100 848 848
 10 10 10 10 32 32 32
 100 100 100 100 100 100 848 848
 10 10 10 10 32 32 32 32
 100 100 100 100 100 100 848 848 1168
 10 10 10 10 10 10 32 32 32 32 32
 100 100 100 100 100 100 100 100 848 848 1168 1168
 10 10 10 10 10 10 10 10 10 10 32 32 32 32 32 32
 100 100 100 100 100 100 100 100 848 848 1168 1168
 10 10 10 10 10 10 32 32 32 32
 100 100 848 848 1168

32

100	848	
10 10 32	10 32	
100 100 848 848	100 100	1168
10 10 10 32 32 32	10 32	10 10 32
100 100 100 100 100 848	100 848	100 100 1168
10 10 10 32 32 32	10 32	10 10 10 10 32 32
100 100 100 100 100 848 848 1168	100 100	100 100 848 848 1168
10 10 10 10 10 10 32 32 32 32	10 10 32	10 10 10 10 32 32
100 100 100 100 100 100 848 1168	100 848 1168	100 100 1168
10 10 10 10 10 10 32 32 32 32	10 10 32	10 10 32
100 100 100 848 848 1168	100 100	1168
10 10 32	10 32	
100	848	

16

100 848	
10 10 16 16* 16*	584
100 100 100 848	10 16
10 10 16 16 16* 16*	100 584*
100 100 100 584* 848	10 10 16* 16*
10 10 10 16 16 16* 16* 16*	100 584 848
100 100 100 100 584 584 848	10 10 16 16
10 10 10 10 10 16 16 16 16 16 16* 16*	100 584*
100 100 100 100 584* 584* 848	10 16*
10 10 10 16 16* 16* 16*	584
100 584 848	

16

3 A_{12} and S_{12} , characteristic 2

This group is getting towards the edge of what can be easily constructed using today's computers. What makes it even more annoying is that there are simple modules in the principal 2-block of A_{12} that are not realizable over \mathbb{F}_2 .

There are eleven simple modules in the principal block of S_{12} , three of which split into two dual modules upon restriction to A_{12} , and they are below.

λ	$\dim(D^\lambda)$	Factors of S^λ	$\dim(P(D^\lambda))$
(12)	1	1	204288
(11, 1)	10	10	159232
(7, 5)	$32 = 16 \oplus 16^*$	32, 164, 100, 1	145408
(10, 2)	44	44, 10	69120
(9, 3)	100	100, 44, 10	116224
(8, 4)	164	164, 100, 10, 1	59904
(6, 5, 1)	$288 = 144 \oplus 144^*$	288, 570, 164, 100, 32, 1	55296
(6, 4, 2)	416	416, 1046, 570, 288, 164, 100, 44, 32, 10, 1^3	50688
(8, 3, 1)	570	570, 164, 100, 44, 10, 1^3	48640
(7, 3, 2)	1046	1046, 570, 164, 100, 32, 10, 1^3	45056
(5, 4, 2, 1)	$2368 = 1184 \oplus 1184^*$	2368, 1046, 570, 416^2 , 288, 164, 100^3 , 44^2 , 32^2 , 10^5 , 1^5	45056

Each of the dual pairs amalgamate over \mathbb{F}_2 . We will not give the structures of the projectives for the 1-, 10-, 16- and 100-dimensional simple modules because it takes too long to find their socle layers. However, we do give the Ext^1 matrix below.

The non-principal block not of defect 0 has 2-core $(3, 2, 1)$, and has three modules as follows for the alternating and symmetric groups.

λ	$\dim(D^\lambda)$	Factors of S^λ	$\dim(P(D^\lambda))$
(9, 2, 1)	320	320	7680
(7, 4, 1)	1408	1408	6656
(5, 4, 3)	1792	1792, 320	5632

There is also a block with defect 1 for S_{12} and of defect zero for A_{12} , with simple of dimension 5632 and given by the partition $(6, 3, 2, 1)$.

Here is the Ext^1 -matrix for the principal block for $p = 2$.

	1	10	16	16*	44	100	144	144*	164	416	570	1046	1184	1184*
1	0	1	0	0	1	0	0	0	1	1	2	1	0	0
10	1	0	0	0	1	2	0	0	0	0	0	1	1	1
16	0	0	0	1	0	1	1	0	0	0	0	1	1	0
16*	0	0	1	0	0	1	0	1	0	0	0	1	0	1
44	1	1	0	0	0	0	0	0	0	0	0	0	0	0
100	0	2	1	1	0	0	0	0	1	0	0	0	0	0
144	0	0	1	0	0	0	0	0	0	1	1	0	0	0
144*	0	0	0	1	0	0	0	0	0	1	1	0	0	0
164	1	0	0	0	0	1	0	0	0	0	0	0	0	0
416	1	0	0	0	0	0	1	1	0	0	0	0	1	1
570	2	0	0	0	0	0	1	1	0	0	0	0	0	0
1046	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1184	0	1	1	0	0	0	0	0	0	1	0	0	0	0
1184*	0	1	0	1	0	0	0	0	0	1	0	0	0	0

We now give the socle layers of the projectives that we have constructed.

44
1
10 164 416 570
1 1 1 1 100 100
10 32 44 44 164 570
1 1 1 100 288 1046 1046
10 10 32 32 44 44 164 164 416 570 570
1 1 1 1 1 1 1 1 10 10 100 100 100 288
10 10 32 44 44 44 100 164 164 164 416 416 570 570 570 2368
1 1 1 1 1 1 1 1 1 1 10 10 10 10 32 100 100 100 100 288 1046
10 10 32 32 44 44 44 44 44 100 100 100 164 164 164 416 416 570 570 1046
1 1 1 1 1 1 1 1 1 1 1 1 1 1 10 10 10 10 10 10 32 32 100 100 100 100 100 288
10 10 32 32 44 44 44 44 100 100 100 100 100 100 164 164 164 164 416 416 416 416 570 570 570 1046
1 1 1 1 1 1 1 1 1 1 1 1 1 1 10 10 10 10 10 10 32 32 100 100 100 100 288 288 288 1046 1046
10 10 10 32 32 32 44 44 44 44 44 100 164 164 164 416 570 570 570 1046 1046 1046 2368 2368
1 1 1 1 1 1 1 1 1 1 1 1 1 1 10 10 10 10 10 10 32 100 100 100 100 288
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10 10 32 32 44 44 44 100 100 100 100 164 164 164 164 416 416 416 416 570 570 570
1 1 1 1 1 1 1 1 1 1 1 1 1 1 10 10 10 10 10 32 100 100 288 1046
10 32 44 44 44 44 100 164 570 1046 2368
1 1 1 1 10 10 10 100 288
44 100 164 416 570
1 10
44

144
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 16 44 164
 1 1 100 144 144* 1046
 10 16 16* 44 164 416 570
 1 1 1 1 100 100 144*
 10 16 16* 44 164 416 570 570
 1 1 1 1 100 144 144* 1046
 10 10 16 16* 16* 44 44 164 164 570 1184*
 1 1 1 1 1 10 100 100 144 144* 1046
 10 16 16* 44 100 100 164 164 416 416 570 570
 1 1 1 1 1 1 10 10 10 16 16* 100 100
 10 16 16 16* 44 44 44 164 570 1046 1184
 1 1 1 1 10 10 100 100 144 144* 1046
 10 16 16* 44 100 164 164 416 570
 1 1 1 1 1 10 100 100 144 144
 10 16 16* 44 164 416 570 570
 1 1 10 16* 100 144 144*
 16 44 164 416 1046
 1 1 144 144* 144* 1184*
 16 416 570
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164
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 10 10 10 32 32 32 44 44 44 100 164 164 164 164 416 416 570 570 570 2368
 1 1 1 1 1 1 1 1 1 1 1 1 10 10 10 10 32 100 100 100 100 288 288 1046
 10 10 32 32 44 44 44 44 100 100 100 164 164 164 164 164 416 416 570 570 570 570 1046
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 10 10 10 10 32 32 32 32 44 44 44 44 44 100 164 164 164 164 164 416 416 570 570 570 1046 2368
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 10 32 44 100 100 100 100 164 164 164 416 416 416 570 570
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 100 100 100 164 416 416 570
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570
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 10 32 44 44 100 164 164 416 570 570
 1 1 1 1 1 1 10 32 100 288 1046
 10 10 32 44 44 44 164 164 416 570 1046
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 10 10 10 32 32 32 44 44 100 164 164 164 416 416 570 570 570 570
 1 1 1 1 1 1 1 1 1 1 10 10 100 100 100 100 288 1046
 10 10 10 32 32 32 44 44 44 44 100 164 164 164 164 416 570 570 570 2368
 1 1 1 1 1 1 1 1 1 10 10 10 10 32 100 100 100 288 288 1046 1046
 10 10 32 32 44 44 44 100 100 100 164 164 164 416 416 570 570 570 1046
 1 1 1 1 1 1 1 1 1 10 10 10 10 32 100 100 100 100 288
 10 10 32 32 44 44 44 100 164 164 164 416 416 570 570 1046 2368
 1 1 1 1 1 1 1 10 10 10 32 100 100 288 1046
 10 32 44 44 44 100 164 164 416 570 570 1046
 1 1 1 1 1 1 1 10 100 100 100 288 288
 10 32 44 164 164 416 570 570
 1 1 1 10 32 100
 44 164 570 1046
 1 1 288
 570

1046
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 1 1 100
 100 164 416 570
 1 1 10 32
 44 44 100 1046
 1 1 10 32 164 288 1046
 1 1 10 10 32 100 164 416 570
 1 1 1 1 100 100 100 164 570 2368
 1 1 10 10 10 10 32 32 44 164 416 570
 1 1 1 44 100 100 100 288 416 1046 1046
 1 1 10 10 10 10 32 32 44 44 100 164 570
 1 1 1 1 10 32 44 44 100 164 288 416 1046 1046 2368
 1 1 1 1 1 10 10 10 10 32 100 100 164 164 416 570 570
 1 1 1 1 10 32 44 100 100 100 100 100 164 416 416 570
 1 1 1 1 10 10 10 10 10 32 32 44 44 100 164 288 1046
 1 1 10 32 44 44 44 100 164 288 570 1046 1046 1046 2368
 1 1 1 1 10 10 10 10 32 32 100 164 416 570
 1 1 1 100 100 100 100 100 164 416 416 570
 1 1 1 10 10 10 10 32 32 44 164
 1 44 100 1046 1046 2368
 1 10 10 32 164 288
 1 100 100 416 570
 1 10 32
 1046

1184
 10
 100
 10
 1184*
 10
 100 416
 1 10 10 16 16
 44 100 1046 1184
 1 1 10 10 16*
 100 164 416 1184
 1 1 10 10 10 16* 16* 144
 44 44 100 100 570 1046 1184*
 1 1 1 10 10 10 16
 100 164 416 416 1184*
 1 1 1 10 10 10 16 16 16 144*
 44 44 100 100 570 1046 1184
 1 1 10 10 10 16*
 100 164 416 1184
 1 1 10 10 16* 16*
 44 100 1046 1184*
 10 10 16
 1 100 144
 10 16 416
 1184

We also have the non-principal block, with projectives as follows:

320	1408	1792
1408 1792	320	320
320 320	1792	1408
1408 1792	320 1408	320
320	1408	1792

The time it took to construct the projectives on my computer is as follows. Note that there are preliminary things like constructing the Cartan matrix and finding good subgroups for condensation that, if one constructs several projectives in one instance of Magma, will not be repeated, so use these numbers as a guide only.

D^λ	time to construct $P(D^\lambda)$ (seconds)
1	64295
10	9196
16 (over \mathbb{F}_4)	27314
32	30555
44	9750
100	14141
164	6570
144 (over \mathbb{F}_4)	2676
288	785
416	7097
570	2328
1046	7647
1184 (over \mathbb{F}_4)	2490
2368	618

4 A_{10} , characteristic 3

In A_{10} in characteristic 3 there are two blocks: the principal block with 3-core (1), of defect 4, and the non-principal block with 3-core (3, 1).

λ	$\dim(D^\lambda)$	Factors of S^λ	λ	$\dim(D^\lambda)$	Factors of S^λ
(10)	1	1	(9, 1)	9	9
(8, 2)	34	34, 1	(6, 4)	90	90
(7, 3)	41	41, 34	(6, 2 ²)	126	126, 90, 9
(7, 2, 1)	84	84, 41, 34, 1	(4 ² , 2)	36	36, 126, 90
(6, 2, 1 ²)	224	224, 84, 41, 1	(3 ² , 2, 1 ²)	279	279, 126, 36, 9

The non-principal block is Morita equivalent to the two non-principal blocks of S_{10} , which are labelled by the 3-cores (3, 1) and its conjugate (2, 1²). These are Morita equivalent to the principal 3-block of S_8 , by standard Scopes moves [4].

The Ext^1 -matrices are easy to describe, and we do this now.

Block 1	1	34	41	84	224
1	0	1	1	1	1
34	1	0	1	0	1
41	1	1	0	1	1
84	1	0	1	1	0
224	1	1	1	0	1

Block 2	9	36	90	126	279
9	0	0	0	1	1
36	0	0	0	1	1
90	0	0	0	1	0
126	1	1	1	0	0
279	1	1	0	0	0

Finally, here are the radical and socle layers of the projectives, which coincide in this case.

1					34				
34 41 84 224					1 41 224				
1 1 1 1 34 41 41 84 224 224					1 34 34 41 84 224				
1 1 1 34 34 34 41 41 41 84 84 224 224					1 1 1 34 41 41 84 224				
1 1 1 1 34 41 41 84 224 224					1 34 34 41 84 224				
34 41 84 224					1 41 224				
1					34				
41					84				
1 34 84 224					1 41 84				
1 1 34 41 41 41 84 224					1 34 41 84 84 224				
1 1 1 34 34 41 84 84 224 224					1 1 34 41 41 84 224				
1 1 34 41 41 41 84 224					1 34 41 84 84 224				
1 34 84 224					1 41 84				
41					84				
9					36				
126 279					126 279				
9 9 36 90					9 36 36 90				
126 279					126 279				
9					36				
90					126				
126					9 36 90				
126 126 279					126 279				
9 36 90					9 36				
126					126				
126					279				

These were described for the symmetric group by Tan in [8], and restriction is simply a case of replacing the module for S_{10} by its restriction.

5 A_{11} , characteristic 3

In A_{11} in characteristic 3 there are two blocks, the principal block with 3-core (2), of defect 4, the non-principal block with 3-core (3, 1, 1) of defect 2, and a block with defect 1 and 3-core (4, 2, 1²).

The principal block of A_{11} is Morita equivalent to that of S_{11} , and we label the modules from this block by 3-regular partitions with 3-core (2), rather than (1²), so that the 10-dimensional simple module has label (5², 1), rather than (10, 1).

λ	$\dim(D^\lambda)$	Factors of S^λ
(11)	1	1
(8, 3)	109	109, 1
(8, 2, 1)	120	120, 109
(7, 3, 1)	320	320, 120, 109, 1
(6, 3, 1 ²)	791	791, 320, 120, 1
(5 ² , 1)	10	10, 320
(5, 4, 1 ²)	34	34, 10, 791, 320, 1 ²
(5, 3 ²)	210	210, 10, 320, 120
(5, 3, 2, 1)	714	714, 210, 34, 10, 791, 320, 120, 109, 1 ²
(4, 3, 2 ²)	131	131, 714, 210, 34, 120, 109, 1 ²

We now give the Ext^1 -matrix, for those readers who do not want to look at the projectives directly.

Block 1	1	10	34	109	120	131	210	320	714	791
1	0	0	0	1	1	1	0	0	1	1
10	0	0	1	0	0	0	1	1	0	0
34	0	1	0	0	0	1	0	0	1	1
109	1	0	0	0	0	0	0	0	0	0
120	1	0	0	0	0	0	1	1	0	0
131	1	0	1	0	0	0	1	0	0	0
210	0	1	0	0	1	1	0	0	1	0
320	0	1	0	0	1	0	0	0	0	1
714	1	0	1	0	0	0	1	0	0	0
791	1	0	1	0	0	0	0	1	0	0

Block 2	45	126	126*	693
45	0	1	1	0
126	1	0	0	1
126*	1	0	0	1
693	0	1	1	0

For those who do, here they are.

1	
109 120 714	
1 1 1	
109 109 120 714	
1 1 1 1 34 210 320	
109 109 120 120 131 714 791	
1 1 1 1 1 34 34 210 210 320	
10 109 109 120 120 131 714 714 791	
1 1 1 1 1 34 210 320	
10 10 109 109 120 120 131 131 714 714 791	
1 1 1 1 1 34 34 210 210 320	
109 120 131 714 791	
1	
	10
	34 210 320
	120 714
	1
	10 10 10 109 131 791
	1 1 34 34 34 210 210 320 320
	10 10 10 120 131 714 791
	34 210 320
	10

		109	120
34		1	1
714		120 714	109
1		1 1	1 210 320
10 109 131 791		109 109	120 120 714
1 1 34 210 320		1 1 34 210 320	1 1
120 714		120 120 714	10 109 109 131 791
1 34 34 210		1 1	1 1 34 210 320
10 10 10 109 120 131 131 714 791	10 109 109 131 791		120 120 714
1 1 34 34 34 210 210 320	1 1 34 210 320		1 1 34 210 320
10 131 714 791	120 714	10 109 120 120 131 791	
34	1		1 210 320
	109		120
	210		320
131	120 714		120
1 34 210	1		1
120 714	10 109 131		10 109 791
1	1 1 34 210 320		1 34 210 320
10 109 131 131	120 714		120 714
1 1 34 34 210 210	1 34 210 210		1 320
10 120 131 131 714	10 10 109 120 131 131 714 791	10 10 109 120 791	
1 34 210	1 1 34 34 210 210 210 320	1 34 210 320 320	
131	10 120 131 714	10 120 791	
	210	320	
714			
1			
109	791		
1 34 210	1 34 320		
120 714	120 714		
1	1		
10 109 131 791	10 109 791		
1 1 34 210 320	1 34 210 320		
120 714 714	10 120 714 791		
1 1 34 210	1 34 320		
10 109 131 714 791	791		
1 34 210			
714			

And the non-principal block with defect 2.

45	126	126*	693
126 126*	45 693	45 693	126 126*
45 693 693	126 126* 126*	126 126 126*	45 45 693
126 126*	45 693	45 693	126 126*
45	126	126*	693

We should note that three of these, of dimensions 109, 120 and 131, were determined by Tan in [7].

6 A_{12} , characteristic 3

And we are back up to the largest case you can feel comfortable with using current computers, and here we do not produce the socle layers of the projective cover of the trivial module.

There are four blocks for A_{12} in characteristic 3, of defects 5, 2, 1 and 0. The block of defect zero has a module of dimension 2673, and the block of defect 1 has two modules of dimensions 891 and 3564.

λ	$\dim(D^\lambda)$	Factors of S^λ
(12)	1	1
(11, 1)	10	10, 1
(10, 1 ²)	45	45, 10
(9, 3)	143	143, 10, 1
(9, 2, 1)	120	120, 143, 45, 10, 1 ²
(8, 4)	131	131, 143, 1
(8, 2 ²)	210	210, 131, 120, 143, 10, 1 ²
(7, 4, 1)	1013	1013, 131, 120, 143, 1
(7, 3, 2)	$126 \oplus 126^*$	252, 1013, 210, 131, 120, 143, 45, 10, 1
(6, 4, 1 ²)	1936	1936, 1013, 120, 10, 1
(6, 3, 2, 1)	$714 \oplus 714^*$	1428, 1936, 252, 1013, 210^2 , 131, 120^2 , 143, 45, 10^2 , 1^4

The principal block has thirteen simple modules. Here we give the Ext^1 matrix for these modules, since the projectives take several pages to write out, and we do not include $P(1)$, although of course this only means that $\text{Ext}^1(1, 1)$ is not given here, and this is ero for all groups G with $O^p(G) = G$.

Block 1	1	10	45	120	126	126*	131	143	210	714	714*	1013	1936
1	0	1	0	1	0	0	1	1	0	1	1	0	1
10	1	0	1	0	0	0	0	0	1	0	0	1	1
45	0	1	0	1	1	1	0	0	0	0	0	0	0
120	1	0	1	0	0	0	0	0	2	0	0	1	0
126	0	0	1	0	0	0	0	0	1	0	0	1	0
126*	0	0	1	0	0	0	0	0	1	0	0	1	0
131	1	0	0	0	0	0	0	1	1	0	0	1	0
143	1	0	0	0	0	0	1	0	0	0	0	0	0
210	0	1	0	2	1	1	1	0	0	1	1	0	0
714	1	0	0	0	0	0	0	0	1	0	0	0	0
714*	1	0	0	0	0	0	0	0	1	0	0	0	0
1013	0	1	0	1	1	1	1	0	0	0	0	0	0
1936	1	1	0	0	0	0	0	0	0	0	0	0	0

We now give the socle layers of the projective modules in the principal block except for the projective cover of the trivial module.

$$\begin{array}{c}
10 \\
1 \ 1936 \\
1 \ 45 \ 143 \ 210 \\
1 \ 10 \ 10 \ 120 \ 120 \ 714 \ 714^* \ 1013 \\
1 \ 1 \ 1 \ 10 \ 120 \ 120 \ 131 \ 714 \ 714^* \\
1 \ 1 \ 1 \ 10 \ 126 \ 126^* \ 131 \ 143 \ 143 \ 143 \ 1936 \\
1 \ 1 \ 1 \ 1 \ 45 \ 126 \ 126^* \ 143 \ 143 \ 210 \ 210 \ 210 \ 1013 \ 1013 \ 1936 \ 1936 \\
1 \ 1 \ 1 \ 10 \ 10 \ 45 \ 45 \ 120 \ 120 \ 120 \ 143 \ 210 \ 210 \ 210 \ 714 \ 714 \ 714^* \ 714^* \ 1013 \\
1 \ 1 \ 1 \ 1 \ 10 \ 10 \ 10 \ 10 \ 120 \ 120 \ 120 \ 131 \ 131 \ 714 \ 714 \ 714^* \ 714^* \\
1 \ 1 \ 1 \ 10 \ 10 \ 120 \ 126 \ 126^* \ 131 \ 143 \ 143 \ 143 \ 1936 \ 1936 \\
1 \ 1 \ 1 \ 1 \ 1 \ 45 \ 45 \ 126 \ 126^* \ 143 \ 143 \ 210 \ 210 \ 210 \ 1013 \ 1936 \\
1 \ 1 \ 10 \ 10 \ 10 \ 10 \ 120 \ 120 \ 120 \ 126 \ 126 \ 126^* \ 126^* \ 131 \ 131 \ 143 \ 210 \ 210 \ 714 \ 714^* \ 1013 \\
1 \ 1 \ 1 \ 1 \ 45 \ 45 \ 45 \ 120 \ 210 \ 210 \ 210 \ 714 \ 714^* \ 1013 \ 1013 \ 1013 \\
1 \ 10 \ 10 \ 10 \ 10 \ 120 \ 120 \ 126 \ 126 \ 126^* \ 126^* \ 131 \ 131 \ 143 \\
1 \ 45 \ 210 \ 1013 \ 1936 \\
10
\end{array}$$

45
 10 120
 1
 126 126* 143
 1 45 210 210 1013 1013
 10 120 120 120 714 714*
 1 1 1 10 10 131 131
 126 126* 143 143 143 1936 1936
 1 1 1 45 45 210 210
 10 10 120 120 126 126* 714 714*
 1 1 1 45 45 210 210 1013 1013
 10 10 10 120 120 120 126 126 126* 126* 131 131 143
 1 45 45 210 210 1013 1013
 10 120 126 126*
 45

 120
 1
 143 1936
 1 1 45 210 210 1013
 10 10 120 120 120 120 120 714 714 714* 714*
 1 1 1 1 1 1 10 10 131 131 131
 126 126 126* 126* 143 143 143 143 143 1936 1936 1936
 1 1 1 1 1 1 1 45 45 45 210 210 210 210 210 210 1013,1013 1013
 10 10 10 120 120 120 120 120 120 143 714 714 714 714 714* 714* 714* 714*
 1 1 1 1 1 1 1 10 10 10 131 131 131 131
 10 120 120 126 126 126* 126* 143 143 143 143 143 1936 1936 1936
 1 1 1 1 1 1 45 45 210 210 210 210 210 210 1013 1013
 10 10 10 120 120 120 120 120 126 126 126* 126* 131 131 143 714 714 714* 714*
 1 1 1 1 1 10 45 45 45 131 210 210 210 210 1013 1013 1013
 10 10 120 120 120 120 126 126 126* 126* 131 131 143 1936
 1 45 210 210 1013
 120

126
 45 210 1013
 10 120 120 714
 1 1 10 131
 126 126* 126* 143 143 1936
 1 1 1 45 210 210 210 1013
 10 120 120 126* 714 714* 714*
 1 1 10 45 131 210 1013
 10 10 120 120 126 126 126 126 126* 131 143 143 1936
 1 1 1 45 45 210 210 210 210 1013 1013
 10 10 120 120 126 126* 126* 126* 131 714
 45 210 1013
 126
 131
 143
 1 1 210 1013
 10 120 120 120 714 714*
 1 1 1 1 10 131 131
 126 126* 143 143 143 143 1936
 1 1 1 1 1 45 45 210 210 210 210 1013
 10 10 120 120 120 120 714 714 714* 714*
 1 1 1 1 10 131 131 131 131
 1 126 126* 143 143 143 210 1013 1936
 1 1 1 10 10 120 120 126 126* 131 131 143 210 210 1013
 1 1 1 45 45 120 210 210 210 714 714* 1013
 1 10 10 120 120 126 126* 131 131 131
 1 143 210 1013
 131

143
 1
 120 714 714*
 1 1 1 10 131
 143 143 143 1936 1936
 1 1 1 1 1 45 210 210 210 1013
 10 10 10 120 120 120 120 120 143 714 714 714 714 714 714
 1 1 1 1 1 1 1 1 10 10 131 131 131 131
 10 120 126 126 126* 126* 143 143 143 143 143 143 1936 1936
 1 1 1 1 1 1 1 1 45 45 45 210 210 210 210 210 210 1013, 1013
 10 10 10 120 120 120 120 120 143 714 714 714 714* 714* 714*
 1 1 1 1 1 1 10 10 131 131 131 1013
 10 120 126 126 126* 126* 131 143 143 143 1936 1936
 1 1 1 1 45 210 210 210 1013
 10 120 143 714 714*
 1 131
 143

 210
 10 120 120 714 714*
 1 1 1 131
 126 126* 143 143 143 1936 1936
 1 1 1 1 1 1 45 45 210 210 210 210 210 1013 1013
 10 10 10 120 120 120 120 120 120 120 714 714 714 714 714* 714* 714* 714*
 1 1 1 1 1 1 1 1 10 10 10 131 131 131 131
 126 126 126 126* 126* 126* 143 143 143 143 143 143 1936 1936, 1936
 1 1 1 1 1 1 1 1 45 45 210 210 210 210 210 210 210 210 1013 1013
 10 10 10 120 120 120 120 120 120 120 126 126 131 714 714 714 714 714* 714* 714* 714*
 1 1 1 1 1 1 1 10 10 45 45 131 131 210 210 210 210 1013 1013
 10 10 10 120 120 120 120 126 126 126 126 126* 126* 126* 126* 131 131 131 143 143 143 1936 1936
 1 1 1 1 45 45 210 210 210 210 210 210 1013 1013
 10 120 120 126 126* 131 714 714*
 210

714
 1
 143 1936
 1 1 210
 10 120 120 714 714 714* 714*
 1 1 1 1 10 131
 126 143 143 143 1936 1936
 1 1 1 1 1 45 210 210 210 210 1013
 10 10 120 120 120 120 714 714 714 714* 714* 714*
 1 1 1 1 1 10 10 131 131
 126 126* 126* 143 143 143 1936 1936
 1 1 1 1 45 210 210 210 210 1013
 10 120 120 714 714 714* 714*
 1 1 10 131
 126 143 1936
 1 210
 714

 1013
 120
 1 10 131
 126 126* 143 1936
 1 1 45 45 210 210 1013
 10 10 120 120 120 714 714*
 1 1 1 10 131
 126 126* 143 143 1936
 1 1 210 210 1013 1013
 10 120 120 126 126* 131 143 714 714*
 1 1 1 10 45 45 131 210 210 1013 1013
 10 10 10 120 120 120 126 126 126* 126* 131 143 1936
 1 1 45 45 210 210 1013 1013
 10 120 126 126* 131
 1013

1936
 1
 10 120 714 714*
 1 1
 143 143 1936
 1 1 1 210 210 1013
 10 120 120 120 714 714 714* 714*
 1 1 1 1 10 10 131
 126 126* 143 143 1936 1936 1936
 1 1 1 1 45 45 210 210 210 1013
 10 10 120 120 120 714 714 714* 714*
 1 1 1 10 131
 126 126* 143 143 1936
 1 1 210 210 1013
 120 714 714*
 1 10
 1936

The non-principal block of defect 2 is much smaller of course, and the projectives are as follows.

54	297	945	1431	1728
1431 1728	1728	1431 1728	54 945	54 945 297
54 54 297 945	54 297 945	54 297 945 945	1431 1728	1431 1728 1728
1431 1728	1728	1431 1728	54 945	54 297 945
54	297	945	1431	1728

7 A_{10} , characteristic 5

Since these can all be understood from the work of Scopes [5] we simply give the structure of the projectives and the dimensions of the D^λ , with the factors of S^λ for the symmetric group. Where the module for the symmetric group splits into two upon restriction to A_n , we note this.

λ	$\dim(D^\lambda)$	Factors of S^λ
(10)	1	1
(9, 1)	8	8, 1
(8, 1 ²)	28	28, 8
(7, 1 ³)	56	56, 28
(6, 1 ⁴)	$35_1 \oplus 35_2$	70, 56
(5 ²)	34	34, 8
(5, 4, 1)	217	34, 28, 8, 1
(5, 3, 1 ²)	$133_1 \oplus 133_2$	266, 217, 56, 28

1	8	28	34	35 _i
8 217	1 28 34	8 56 217	8 217 217	56
1 1 28 34 34	8 8 217	1 28 28 34 133 ₁ 133 ₂	1 1 28 34 34 34 133 ₁ 133 ₂	35 _{3-i} 133 _i
8 217	1 28 34	8 56 217	8 217 217	56
1	8	28	34	35 _i
56		133 _i	217	
28 35 ₁ 35 ₂ 133 ₁ 133 ₂	56 217		1 28 34 34 133 ₁ 133 ₂	
56 56 56 217	28 34 35 _i 133 ₁ 133 ₂	8 56 217 217 217		
28 35 ₁ 35 ₂ 133 ₁ 133 ₂	56 217	1 28 34 34 133 ₁ 133 ₂		
56	133 _i	217		

8 A_{11} , characteristic 5

Here two modules for S_{11} from the principal block do not remain irreducible when restricted. One restricts to two self-dual modules and one restricts to two dual modules, as we see below.

λ	$\dim(D^\lambda)$	Factors of S^λ
(11)	1	1
(9, 2)	43	43, 1
(8, 2, 1)	188	188, 43
(7, 2, 1 ²)	406	406, 188
(6, 5)	89	89, 43
(6, 4, 1)	372	372, 89, 188, 43, 1
(6, 3, 1 ²)	133 ₁ \oplus 133 ₂	266, 372, 406, 188
(6, 2, 1 ³)	126 \oplus 126*	252, 266, 406

1	43	89	126	133 ₁
43 372	1 89 188	43 372 372	133 ₂	126 372 406
1 1 89 89 188	43 43 372	1 1 89 89 89 133 ₂ 133 ₁ 188	126* 406	89 133 ₁ 133 ₂ 133 ₂ 188
43 372	1 89 188	43 372 372	133 ₁	126* 372 406
1	43	89	126	133 ₁
188		372	406	
43 372 406	1 89 89 133 ₂ 133 ₁ 188	133 ₂ 133 ₁ 188		
1 89 133 ₂ 133 ₁ 188 188	43 372 372 372 406	126 ₁ 126 ₂ 372 406		
43 372 406	1 89 89 133 ₂ 133 ₁ 188	133 ₂ 133 ₁ 188		
188	372	406		

9 A_{12} , characteristic 5

Here the principal blocks of A_{12} and S_{12} are Morita equivalent, and the non-principal block of S_{12} of full defect is dual to it. We give the projectives and the factors of S^λ for the principal block only.

λ	$\dim(D^\lambda)$	Factors of S^λ
(12)	1	1
(9, 3)	153	153, 1
(8, 3, 1)	738	738, 153
(7, 5)	144	144, 153
(7, 4, 1)	372	372, 144, 738, 153, 1
(7, 3, 1 ²)	1266	1266, 372, 738
(7, 2, 1 ³)	462	462, 1266
(6, 3, 1 ³)	1596	1596, 462, 1266, 372
(5, 3, 1 ⁴)	1506	1506, 1596, 462
(4 ³)	89	89, 372, 1
(4, 3 ² , 1 ²)	1957	1957, 89, 1596, 372, 144
(4, 3, 2, 1 ³)	573	573, 1957, 1506, 1596
(3 ³ , 2, 1)	11	11, 1957, 144
(3 ² , 2 ² , 1 ²)	43	43, 11, 573, 1957, 89

1	11	43	89
153 372	43 1957	11 89 573	43 372 1957
1 1 89 144	11 11 89 144 573	43 43 1957	1 11 89 89 144 573 1596
153 372	43 1957	11 89 573	43 372 1957
1	11	43	89
144	153	372	
153 372 1957	1 144 738	1 89 144 738 1266 1596	
1 11 89 144 144 738 1596	153 153 372	153 372 372 372 462 1957	
153 372 1957	1 144 738	1 89 144 738 1266 1596	
144	153	372	
462	573	738	1266
1266 1596	43 1506 1957	153 372	372 462
372 462 462 1506	11 89 573 573 1596	1 144 738 1266	738 1266 1596
1266 1596	43 1506 1957	153 372	372 462
462	573	738	1266
1506	1596	1957	
573 1596	372 462 1506 1957	11 89 144 573 1596	
462 1506 1957	89 144 573 1266 1596 1596	43 372 1506 1957 1957	
573 1596	372 462 1506 1957	11 89 144 573 1596	
1506	1596	1957	

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